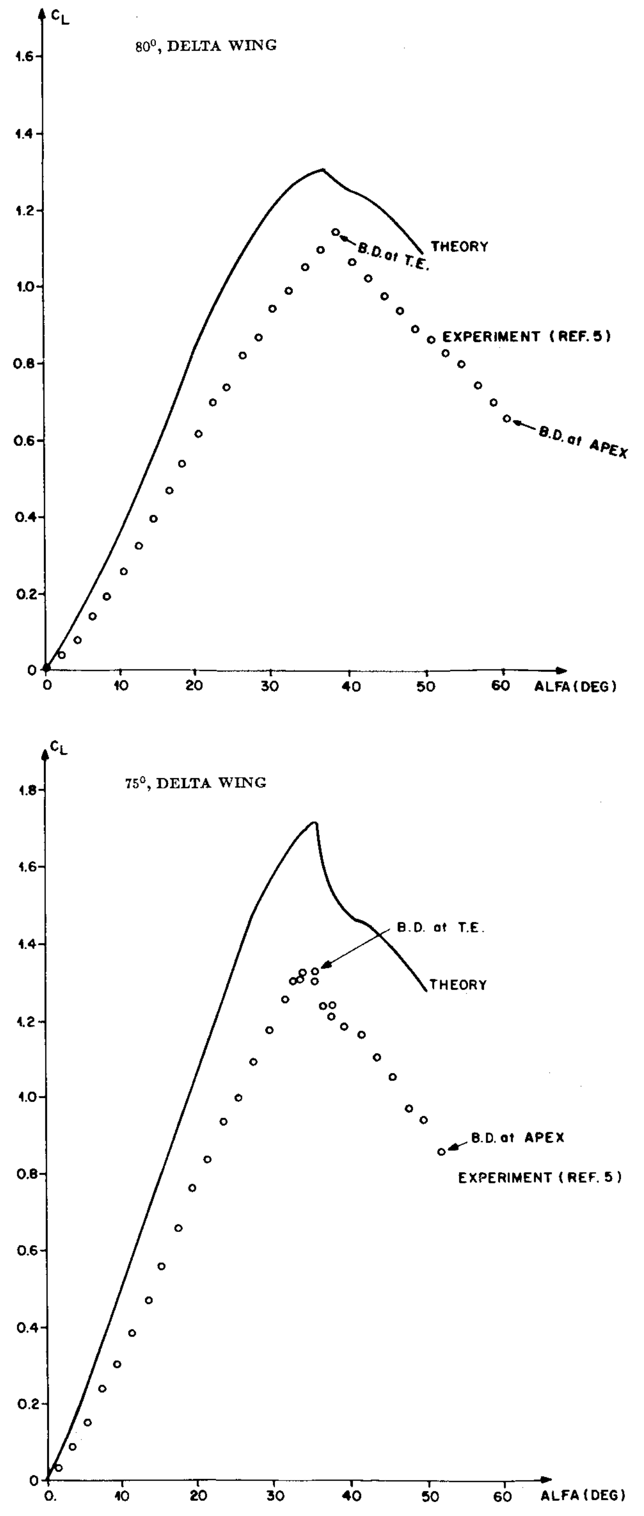

Fig. 3 Influence of the breakdown on lift coefficient.

three-dimensional characteristics, should improve the agreement between the present method and the experimental results.

Acknowledgment

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Baldwin-Lomax Factors for Turbulent Boundary Layers in Pressure Gradients

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Introduction

THE Baldwin-Lomax relations[1] for eddy viscosity have become advantageous for solving the Reynolds-averaged Navier-Stokes equations for turbulent shear flows.[2,3] Baldwin and Lomax modified the Cebeci-Smith algebraic model by substituting new relations for conditions at the outer edge of the shear flow. The numerical solution was facilitated by eliminating the need to determine the displacement thickness and the boundary-layer thickness. To accomplish this, two factors, C_{kleb} and C_{cp}, were introduced and given constant values. York and Knight[4] used other constant values for C_{kleb} and C_{cp} when applying the Baldwin-Lomax model to nonseparating, incompressible (low-Mach number) turbulent boundary layers in pressure gradients. Even when using constant values of C_{kleb} and C_{cp}, York and Knight indicated a variation with the Coles wake factor. However, the extent of the variation was not given. Accordingly, this technical note derives relations that show a large variation of Baldwin-Lomax factors C_{kleb} and C_{cp} with the Coles wake factor representing favorable pressure gradients as well as adverse pressure gradients up to separation. The basis is the well-substantiated outer similarity velocity law. By eliminating the Coles wake factor a formula is derived for C_{cp} as a function of C_{kleb}. For the case of equilibrium boundary layers, the Baldwin-Lomax factors are given as functions of a modified Clauser pressure-gradient parameter. Finally, the variation of the Clauser factor (k) with low Reynolds number is adapted to the Baldwin-Lomax model.

Baldwin-Lomax Model

For the outer region, which may encompass most of the boundary layer, the kinematic eddy viscosity (ν_t) in the Baldwin-Lomax model may be written, in their rather odd notation, for turbulent boundary layers as

$$\nu_t = kC_{cp}F_{\text{wake}}\,[1+5.5(C_{\text{kleb}}y/y_{\max})^6]^{-1} \quad (1)$$

$$F_{\text{wake}} = y_{\max}F_{\max} \quad (2)$$

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$$F_{\max} = \max\left[y\left|\frac{du}{dy}\right|D\right] \tag{3}$$

where y is the normal distance from the wall, $y_{\max}$ the value of y corresponding to $F_{\max}$, k the Clauser factor, u the streamwise flow velocity, and D is the van Driest damping function, which is equal to one for the outer region for the case here. The corresponding Cebeci-Smith model is

$$\nu_t = kU\delta^*[1+5.5(y/\delta)^6]^{-1} \tag{4}$$

where U is the flow velocity outside the boundary layer, δ^* the displacement thickness, and δ the boundary-layer thickness. Note that U, δ^*, and δ have been eliminated in the Baldwin-Lomax model.

Baldwin-Lomax Factors C_{kleb} and C_{cp}

The variation of the Baldwin-Lomax factors C_{kleb} and C_{cp} may be obtained as follows. The outer similarity law[5] for turbulent boundary layers in pressure gradients, equilibrium or nonequilibrium, on smooth or rough surfaces is given by

$$\frac{U-u}{u_\tau} = -A\ell n\left(\frac{y}{\delta}\right) + B_2\left[1-3\left(\frac{y}{\delta}\right)^2+2\left(\frac{y}{\delta}\right)^3\right]$$
$$-A\left[\left(\frac{y}{\delta}\right)^2-\left(\frac{y}{\delta}\right)^3\right] \tag{5}$$

where the last term ensures that $du/dy=0$ at the outer edge of the boundary layer and the next-to-last term is the Coles wake function given as a polynomial by Moses.[6] The slope of the logarithmic law and the reciprocal of the von Kármán constant is A, and B_2 is the outer-law logarithmic intercept. Also, by definition, $B_2 = 2A\Pi$ where Π is the Coles wake factor; B_2 or Π may vary with the pressure gradient, u_τ is the shear velocity, $u_\tau = \sqrt{\tau_w/\rho}$ where τ_w is the wall shear stress, and ρ is the fluid density. Consequently,

$$y\left|\frac{du}{dy}\right| = u_\tau\left\{A\left[1+2\left(\frac{y}{\delta}\right)^2-3\left(\frac{y}{\delta}\right)^3\right]\right.$$
$$\left.+6B_2\left[\left(\frac{y}{\delta}\right)^2-\left(\frac{y}{\delta}\right)^3\right]\right\} \tag{6}$$

Differentiating to obtain the maximum yields

$$C_{\text{kleb}} = \frac{4(A+3B_2)}{9(A+2B_2)} = \frac{4(1+6\Pi)}{9(1+4\Pi)} \tag{7}$$

and

$$F_{\max} = u_\tau \tilde{F}_{\max} \tag{8}$$

where

$$\tilde{F}_{\max} = A[1+2C^2_{\text{kleb}} - 3C^3_{\text{kleb}} + 12\Pi(C^2_{\text{kleb}} - C^3_{\text{kleb}})] \tag{9}$$

Since $y_{\max} = C_{\text{kleb}}\delta$ and $\delta/\delta^* = U/u_\tau I_1$, where

$$I_1 = \int_0^1 [(U-u)/u_\tau]\,d(y/\delta) = A[(11/12)+\Pi]$$

and if the outer law [Eq. (5)] is assumed to hold up to the wall $y=0$, then

$$F_{\text{wake}} = \frac{\tilde{F}_{\max} C_{\text{kleb}} U\delta^*}{I_1} \tag{10}$$

A comparison of the Baldwin-Lomax relation [Eq. (1)] to the Cebeci-Smith relation [Eq. (4)] for eddy viscosity shows that

$$U\delta^* = C_{cp}F_{\text{wake}} = \left(\frac{C_{cp}\tilde{F}_{\max}C_{\text{kleb}}}{I_1}\right)U\delta^* \tag{11}$$

or

$$C_{cp} = \frac{I_1}{\tilde{F}_{\max}C_{\text{kleb}}} \tag{12}$$

Then, from Eq. (7),

$$\Pi = \frac{4-9C_{\text{kleb}}}{12(3C_{\text{kleb}}-2)} \tag{13}$$

$$C_{cp} = \frac{3-4C_{\text{kleb}}}{2C_{\text{kleb}}(2-3C_{\text{kleb}}+C^3_{\text{kleb}})} \tag{14}$$

At separation, the Coles wake factor $(\Pi)\to\infty$, so that from Eq. (7), $C_{\text{kleb}} = 2/3$ and from Eq. (14), $C_{cp} = 27/32$.

The variation of the Baldwin-Lomax factors C_{kleb} and C_{cp} with the Coles wake factor Π is shown in Fig. 1. The condition of zero pressure gradient (flat plate) is given by $A=2.5$, $B_2=2.4$, and consequently $\Pi=0.48$. The large variation for favorable pressure gradients is to be noted. The variation of C_{cp} as a function of C_{kleb} according to Eq. (14) is shown in Fig. 2. The constant values used by York and Knight, $C_{cp}=1.2$ and $C_{\text{kleb}}=0.646$, are also shown.

Equilibrium Pressure Gradients

For the case of equilibrium pressure gradients, the specified values of B_2 or Π remain constant in the streamwise direction. Also, the Rotta-Clauser shape parameter G remains constant: G is given as a function of Π as follows. By definition, $G = I_2/I_1$ where

$$I_2 = \int_0^1 [(U-u)/u_\tau]^2\,d(y/\delta) \tag{15}$$

and hence from Eq. (5)

$$I_2 = A^2\left(\frac{4819}{2520} + \frac{213}{70}\Pi + \frac{52}{35}\Pi^2\right) \tag{16}$$

$$G = A\left(\frac{4819}{2520} + \frac{213}{70}\Pi + \frac{52}{35}\Pi^2\right)\Big/\frac{11}{12}+\Pi \tag{17}$$

Nash[7] has empirically correlated G to the Clauser pressure-gradient parameter β where $\beta = (\delta^*/\tau_w)(dp/dx) = -(U\delta^*/u_\tau^2)(dU/dx)$, p is the pressure, and x the streamwise coordinate, such that

$$G = 6.1\sqrt{1.81+\beta} - 1.56 \tag{18}$$

The last term has been slightly altered so that from $A=2.5$, $\Pi=0.48$, $G=6.65$ for a zero pressure gradient, $\beta=0$.

The Clauser pressure-gradient parameter β is fitted to the Baldwin-Lomax model so that

$$\beta = \left(\frac{I_1}{C_{\text{kleb}}}\right)\frac{y_{\max}}{u_\tau}\frac{dU}{dx} \tag{19}$$

Since I_1 is a function of C_{kleb} from Eq. (13) or

$$I_1 = \frac{A}{2}\left(\frac{4C_{\text{kleb}}-3}{3C_{\text{kleb}}-2}\right) \tag{20}$$

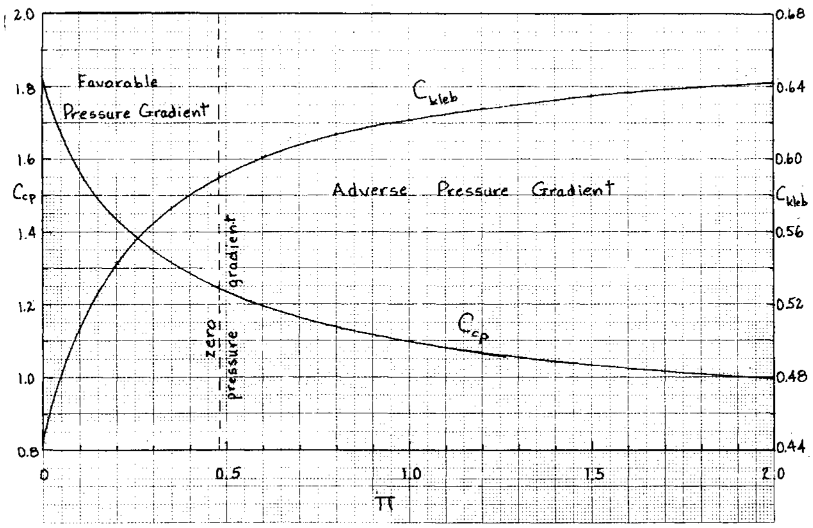

Fig. 1 Variation of C_{cp} and C_{kleb} with Coles wake factor Π.

and C_{kleb} through G is a function of β, a modified Clauser pressure-gradient parameter $\tilde{\beta}$ is proposed which is evaluated from

$$\tilde{\beta}=\frac{y_{max}}{u_\tau}\frac{dU}{dx} \tag{21}$$

The variation of C_{kleb} with $\tilde{\beta}$ is then given implicitly by

$$\tilde{\beta}=\left[\left(\frac{G+1.56}{6.1}\right)^2-1.81\right]\frac{2C_{kleb}(3C_{kleb}-2)}{A(4C_{kleb}-3)} \tag{22}$$

and from Eqs. (13) and (17):

$$G=\frac{A(4704C_{kleb}^2-6755C_{kleb}+2430)}{210(4C_{kleb}-3)(3C_{kleb}-2)} \tag{23}$$

Plots of C_{kleb} and C_{cp} are shown as functions of $\tilde{\beta}$ in Fig. 3.

After a numerical analysis, the variation of C_{kleb} with $\tilde{\beta}$ may be given explicitly by

$$C_{kleb}=\frac{2}{3}-\frac{0.01312}{0.1724+\tilde{\beta}} \tag{24}$$

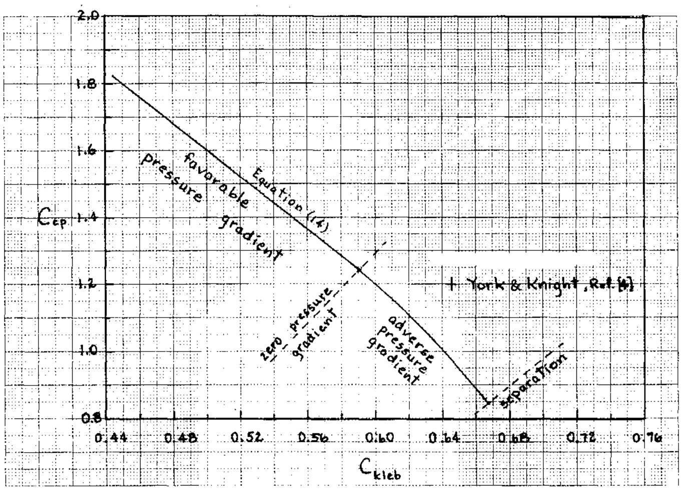

Fig. 2 Variation of C_{cp} with C_{kleb}.

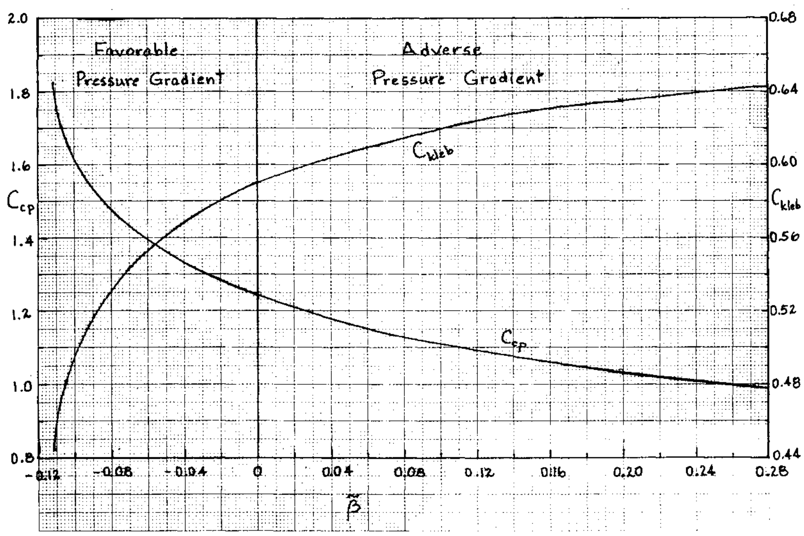

Fig. 3 Variation of C_{cp} with C_{kleb} with modified Clauser pressure- gradient parameter $\tilde{\beta}$.

Low Reynolds Number

For the low Reynolds number region following the transition to turbulent flow, the proposal of Herring and Mellor (cited in Ref. 8) that the Clauser factor k varies with the displacement-thickness Reynolds number, $U\delta^*/\nu$ may be written as

$$k = k_\infty [1 + (1100\nu / U\delta^*)^2] \quad (25)$$

where ν is the kinematic viscosity of the fluid and k_∞ is the value of k at high Reynolds number. Cebeci and Smith use a value of $k_\infty = 0.0168$. From Eq. (11), where $U\delta^* = C_{cp} F_{\text{wake}}$, the Baldwin-Lomax model can be written as

$$k = 0.0168 [1 + (1100\nu / C_{cp} F_{\text{wake}})^2] \quad (26)$$

Concluding Remarks

The pertinent formulas are given by Eq. (24) for the variation of C_{kleb} with the modified Clauser pressure-gradient parameter $\bar{\beta}$, by Eq. (14) for the variation of C_{cp} with C_{kleb}, and by Eq. (26) for the variation of the Clauser factor k with low Reynolds number. As presented here, the relations for the Baldwin-Lomax factors have an indirect experimental validity by being derived from the experimentally confirmed velocity similarity laws. Finally, it should be apparent that for boundary layers in pressure gradients, constant values for the Baldwin-Lomax factors are unsatisfactory in satisfying the velocity similarity laws. To approach the same accuracy as the parent Cebeci-Smith method, the Baldwin-Lomax factors should be varied as indicated.

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Modification of the Osher Upwind Scheme for Use in Three Dimensions

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Introduction

THE Osher upwind scheme, developed for the solution of hyperbolic conservation laws, is an upwind, shock-capturing algorithm based on an approximate Riemann solver. Unlike an exact Riemann solver, it uses compression waves to approximate shocks and thereby simplifies the algorithm.[1,2] In the solution of the Euler equations in one and two dimensions, this scheme is shown to capture strong shocks crisply with transition over two grid points when the grid is aligned with the shock. The scheme was originally introduced as first-order accurate in space and time with explicit time differencing. However, for two-dimensional problems, Chakravarthy and Osher[2] present a second-order accurate version, while Rai and Chakravarthy[3] present an implicit form of the scheme.

Although the scheme has been used extensively in two dimensions, it has been outlined only for three-dimensional use in Ref. 4. As presented in this reference the algorithm is unsuitable for general computations in three dimensions, however, since it requires that the metrics ξ_x, η_x, and ζ_x of the grid transformation are nowhere zero. Two forms of a necessary modification of the scheme for use on a three-dimensional arbitrary grid are described in this Note.

Need for Modification of the Three-Dimensional Osher Scheme

For a complete description of the Osher upwind scheme, the reader is referred to Refs. 1, 2, and 4. The notation in the Note follows that used in these references.

Key to the development of the Osher algorithm is the identification of invariant quantities, generalized Riemann invariants, along each subpath in state space between adjacent grid points (see the schematic in Fig. 1). By definition, Riemann invariants, Ψ, associated with the nth eigenvalue must satisfy

$$\nabla_Q \Psi \cdot r_n(Q) = 0 \quad (1)$$

where r_n is the right eigenvector associated with the nth eigenvalue of the flux Jacobian matrices and Q the vector of dependent variables. It may be shown that, given the above definition, the Ψ quantities are invariant along their distinctive subpath.[1] Equation (1) is the only condition necessary to qualify a Riemann invariant.

Figure 1 presents Riemann invariants suggested by Ref. 4 for three-dimensional computations using the Osher scheme. Equating these invariants along their respective subpaths with known values at grid points (for example, $m-1$ and m) produces 10 equations for the 10 unknown quantities at in-

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